SOLUTION TO DESCRIBE THE INTERNAL KINEMATICS IN BALL BEARINGS WITH 2, 3 OR 4 CONTACT POINTS

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ABSTRACT.

The 3 or 4 contact point ball bearings, have the special inner and / or outer rings geometry. Such bearings may operate smoothly from 2 to 3 or 4 contact points while changing operating conditions. For ball bearings with up to 2 point contacts, the control criteria of ball bearing under the inner or outer raceway is unusable. The paper presents a mathematical model to describe the ball internal kinematics under the effect of the external working conditions. A computer code named SRB-4PCBB was developed to 2, 3 and 4 point contact ball bearings analysis. The model analyses the internal kinematics of the ball by the principle of power minimization taking into account the gyroscopic and spin movement effect in the ball equilibrium. Because in this paper is presented only the quasi-static equilibrium, the cage and ball angular speed are taken as constants values

KEYWORDS: arched race ball bearings, internal kinematics, 2 to 4 point contact ball bearings, spin and gyroscopic movement, ball equilibrium.

1. INTRODUCTION

In the ball bearings with arched-outner-race the centrifugal forces is shared between the two ball-outer race contacts above some transition speed. Therefore, when the arched bearing has two contact points per ball at the outer race, the centrifugal loading can be shared and thereby reduce the maximum pressure at the outer ring contacts that subsequently will increase the bearing life. A first analysis of an arched bearing design was performed by Hamrock and Anderson [1] who indicated the possibility of significant fatigue life improvement. They developed an quasi-static analysis of an arched-outner race ball bearing considering only ball centrifugal forces, neglecting gyroscopic. The model was later improved [2] by adding the effect of the gyroscopic moment acting on the balls.

Recent investigations on high-speed lubricated ball bearings have revealed that the power loss by ball spinning is shared between the inner and the outer race [3]. The present paper proposes a mathematical model to describe the ball equilibrium of ball bearings with 2, 3 or 4 contact points. The bearing element kinematics has been solved considering a mixed control of the ball between the bearing raceways. The main mathematical considerations concerning the computing method are presented in [3].

2. GEOMETRY CONSIDERATIONS

The model has been developed imposing the outer ring(s) fixed in space. Figure 1 presents a simplified cross-section of a four points contact ball bearing (4PCBB), pointing out the position of the raceway curvature centers (points Pidx) relative to the ball mass center (point Ow). The co-ordinate system, external load vector \( \{E\} \) and deformation vector \( \{\delta\} \) are presented in figure 2. The definition of each symbol is given in the nomenclature.

Fig. 1. Schematic view of a 4 points contact ball bearing (4PCBB).
Under external applied loads or imposed rotations and translations of the inner ring the raceway curvature centers reach their final positions noted $P_{d_{idx}}$, as schematically presented in figure 3. For each ball numbered by $j=1$ to $Z$, the nominal and operating contact angles, respectively $\alpha_{idos}$ and $\beta_{idos}$, have their origin coincident with the OZ-axis.

**QUASI-STATIC ANALYSIS CONSIDERATIONS**

Under the effect of the external load the inner ring(s) of the 4PCBB structure is (are) displaced. This displacement produce at the ball-raceways interfaces a load distribution. The normal load and the elastic deflection of each point contact between ball "$j$" and raceway "idx" are given according with Hertz theory [5].

$$Q_{j, idx} = K_{idx} \delta_{j, idx}$$

where:
- $K_{idx}$ represents the contact stiffness.
- $\delta_{j, idx}$ represents the elastic deflection in the ball / ring contact.

Ball equilibrium

Using the significance term for $sdx$, $sdz$, $sdux$, $sduz$ presented in Table 1 and [3,5], two equilibrium equations are written for each ball according to Eq.1. The sign effect of the gyroscopic movement is introduced by using the $smgz$, $smgx$ coefficients presented in [3, 5].

$$\sum Q_{j, idx} \left[ \begin{array}{c} \sin(\beta_{j, idx}) \cdot sdx_{idx} \\ -\cos(\beta_{j, idx}) \cdot smgx_{idx} \cdot CFB_j(\beta_w) \end{array} \right] = 0$$

$$\sum Q_{j, idx} \left[ \begin{array}{c} +\sin(\beta_{j, idx}) \cdot smgz_{idx} \cdot CFB_j(\beta_w) \\ \cos(\beta_{j, idx}) \cdot sduz_{idx} \end{array} \right] + F_c = 0$$

with:
- $Q_{j, idx} = Q_{j, idx}(ux, uz)$ = contact load
- $\beta_{j, idx} = \beta_{j, idx}(ux, uz)$ = contact angles, according to [4].
- $\beta_w$ – the ball angle

The Newton-Raphson iterative method is applied to find the new components of the ball mass center displacements ($ux$, $uz$), as function of the components of the previous vector $\delta$.

The $CFB_j(\beta_w)$ functions are:

$$CFB_j(\beta_w) = \frac{2}{\omega_c \omega_b} \sum_{idx} Q_{j, idx} \left[ \begin{array}{c} \frac{\sin(\beta_{j, idx})}{\omega_c \omega_b} \cdot sdx_{idx} \\ -\frac{\cos(\beta_{j, idx})}{\omega_c \omega_b} \cdot smgx_{idx} \cdot CFB_j(\beta_w) \end{array} \right]$$

where:
- $M_{gyr}(\beta_w) = \rho_{ball} \cdot Dw^5 \cdot \alpha_x \cdot \omega_c \cdot \omega_b \cdot |\sin(\beta_w)| \cdot \frac{\pi}{60}$

The $\delta_{idos}$ coefficient results by following the next algorithm:

$$\sum_{idx} \delta_{idos} = \lambda = 1$$
\[ \lambda M_{gyr}(\beta w) = M_{gyr}(\beta w) = CFB(j, \beta w) \frac{dw}{2} \sum_j Q_{j, idx} \]

\[ \lambda_{idx} M_{gyr}(\beta w) = CFB(j, \beta w) \frac{dw}{2} Q_{j, idx} \]

By summation
\[ M_{gyr}(\beta w) \sum_{idx} \lambda_{idx} = CFB(j, \beta w) \frac{dw}{2} \sum_{idx} Q_{j, idx} \]

\[ M_{gyr}(\beta w) = CFB(j, \beta w) \frac{dw}{2} \sum_{idx} Q_{j, idx} \]

**Pseudo code**

initial solution
\[ CFB \leftarrow 0; \beta w \leftarrow 0 \]
Solve Eq. 1 : ECFA=0; ECFR=0; \[ \Rightarrow \ Q_{idx}, \beta w \]
REPEAT
\[ Q_0_{idx} \leftarrow Q_{idx} \]
Compute Eq.2 \[ \Rightarrow CFB(\beta w, Q_{idx}) \]
Solve Eq.1 => \[ Q_{idx}(CFB) \]
UNTIL \[ |Q_{idx} - Q_0_{idx}| < \epsilon \]

**INTERNAL KINEMATICS**

Assumption of mixed control of the ball

Figure 4 shows the necessity to develop a new mathematical model because the inner or outer raceway control cannot be assumed for all ball bearings types.

**Fig. 4. Mixed ball bearing control necessity**

**Sliding speeds**

The speed vector at point P of the contact ellipse \( V_p \) has three components as shown in Fig. 6:
\[ \dot{V}_p = V_{sp} + \dot{V}_p + \dot{V}_{sp} \]  \( \text{(4)} \)

where \( V_{sp} \) is the linear sliding velocity due to the spin movement \( \dot{V}_{sp} = \omega_0 \cdot r \) with \( \omega_0 \), the spin angular rotational speed and \( r \) the distance from the ellipse center; \( \dot{V}_p \) is the linear speed in the rolling direction; \( \dot{V}_{sp} \) is the linear speed in the transverse direction.

The present model uses this hypothesis and the criteria of minimum power losses around the ball to deduce the direction of the ball angular rotational speed vector (defined by angle \( \beta w \), in figure 5) \[ \text{[4, 6]} \].

The hypothesis of minimum power dissipated by ball "\( i \)" is written as \[ \text{[3, 5]} \]:
\[ \frac{\partial \dot{V}_p}{\partial P_x} \frac{\partial P_x}{\partial P_w} \frac{\partial P_y}{\partial P_w} = 0 \]  \( \text{(5)} \)

where:
\( P_x \) represents the power loss due to sliding along the rolling direction;
\( P_y \) represents the power loss due to sliding along the transverse (axial) direction.

\[ \beta w \]

**Fig. 5 – Contact angles \( \beta_1, \beta_2, \beta_3, \beta_4 \) and ball axis attitude angle \( \beta_w \).**

**Fig. 6 – Linear speed components for point P.**

Assuming that the friction coefficient is uniform over the contact area eq. (5) becomes:
\[ \left[ \text{[6]} \right] \]

where:
\[ A_{idx, \xi} = \left[ B_{e c h, \xi} \cdot a_{idx} = \left[ \frac{Dw}{2} \right] \cdot a_{idx} \right] \]

\[ r_{idx} = \frac{Dw \cdot f_{idx}}{Dw} \]

\[ u_{idx} = [-\beta_1, \beta_2, -\beta_3, \beta_4] \]

\[ \omega_{idx} = \alpha \cos(u_{idx} - \beta_w) \]

\( V_{sp} \) is the linear speed of point \( P \) on ball \( "i" \), function of parameters \( (j, idx, \xi) \):
\[ \{V_{sp, \xi, idx}\} = A_{idx, \xi} + \xi \cdot \omega_{idx} \cdot s_d u_{z, idx} \]

\( V_{p} \) is the linear speed of point one raceway "i" function of parameters \( (j, \xi) \):
\[ \{V_{p, \xi}\} = \left[ \frac{dm}{2} \cdot s_d u_{z, \xi} (A_{idx, \xi} \cdot \cos(\beta_{j, idx}) - \xi \cdot \sin(\beta_{j, idx} \cdot s_d u_{z, idx})) \right] \cdot \left[ a_{idx, \xi} \cdot \omega_{idx} \right] \]

The sliding velocity for point \( P(\xi) \) of contact "\( \xi \)" is the difference of the absolute velocities:
\[ \{Vsli\}_{idx} = \{V_{Gr, idx} - \{Vw_{P, idx}\}\} \cdot T_{sdu, idx} \]

where: \( T_{sdu, idx} = 0 \) if \( Q_{idx} = 0 \); or \( T_{sdu, idx} = 1 \) if \( Q_{idx} > 0 \).

If the contact load respects the Hertz theory, \( Q_{idx} \) and \( q_{idx} \) are given as:

\[ Q_{idx} = \sum_{a} Q_{a} \cdot \frac{a_{idx} \cdot b_{idx}}{\pi \cdot a_{idx} \cdot b_{idx}} \]

\[ q_{idx} = \frac{Q_{idx}}{4 \pi} \cdot \frac{1}{1 + \frac{1}{Q_{idx}}} \]

If non–Hertzian contact occur, \( Q_{idx} \) and \( q_{idx} \) are given according with [3, 6], as:

\[ q_{idx} = E_{0} \cdot k_{-0.11} \cdot \Delta_{l} \cdot f \cdot \frac{Q(k_{idx}, \xi)}{0.94896 - 0.09445 \cdot \ln(k_{idx}, \xi)} \]

Finally eq. (6) gives the ball axis attitude angle \( \beta_w \).

### APPLICATIONS

The mathematical model is applied to 4PCBB-1234, 4PCBB-134, 4PCBB-123 and 4PCBB-13 bearing types by imposing constants values for cage and ball speeds, and also for the main ball bearings parameters. The modified parameters are the axial and radial displacement of the inner ring(s). The input data are presented in table 2. Four analyses (A1, A2, A3, A4) are presented in what follows.

For all tests the angular speeds for the cage (\( \omega_{c} \)) and for the balls are given as:

\[ \omega_{c} = \omega_{i} \cdot (1 - \gamma), \quad \omega_{b} = \omega_{i} \cdot (1 - \gamma^2) \cdot \frac{D_{m}}{D_{w}} / 2 \]

\( \omega_{i} = \frac{n_{i} \cdot \pi}{30} \) and \( \gamma = \frac{D_{w}}{D_{m}} \).

The results are presented in table 3, as follows.

<table>
<thead>
<tr>
<th>Application example</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing type</td>
<td>4PCBB-1234</td>
<td>4PCBB-134</td>
<td>4PCBB-123</td>
<td>4PCBB-13</td>
</tr>
<tr>
<td>Ball diameter, ( D_{w} ) (mm)</td>
<td>20</td>
<td>150</td>
<td>0.525</td>
<td>0.51</td>
</tr>
<tr>
<td>Bearing pitch diameter, ( D_{m} ) (mm)</td>
<td>20</td>
<td>20</td>
<td>0</td>
<td>20 (contact angle)</td>
</tr>
<tr>
<td>Inner ring curvature factor, ( f_{i} )</td>
<td>20</td>
<td>20</td>
<td>0</td>
<td>20 (contact angle)</td>
</tr>
<tr>
<td>Outer ring curvature factor, ( f_{e} )</td>
<td>0.525</td>
<td>0.51</td>
<td>0</td>
<td>0 (contact angle)</td>
</tr>
<tr>
<td>Inner ring shim angle (deg)</td>
<td>20</td>
<td>0</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>Outer ring shim angle (deg)</td>
<td>30</td>
<td>30</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Angular speeds, rpm</td>
<td>( n_{i} = 1000 )</td>
<td>( n_{i} = 20000 )</td>
<td>( n_{i} = 1000 )</td>
<td>( n_{i} = 20000 )</td>
</tr>
<tr>
<td>Radial displacement of the IR, mm</td>
<td>( d_{z} = 0.08 )</td>
<td>( d_{z} = 0.03 ) and ( d_{z} = 0 )</td>
<td>( d_{z} = 0.05 )</td>
<td>( d_{z} = 0.03 ) and ( d_{z} = 0 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test type</th>
<th>Contact load, N</th>
<th>Contact angle, deg</th>
<th>Inclination angle of the ball angular rotational speed, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEST A1</td>
<td>( dx )</td>
<td>id( x = 1 )</td>
<td>id( x = 2 )</td>
</tr>
<tr>
<td>-0.05</td>
<td>6359</td>
<td>14516</td>
<td>7863</td>
</tr>
<tr>
<td>-0.03</td>
<td>7672</td>
<td>12556</td>
<td>8777</td>
</tr>
<tr>
<td>0</td>
<td>9932</td>
<td>9932</td>
<td>10437</td>
</tr>
<tr>
<td>TEST A2</td>
<td>( dx )</td>
<td>id( x = 1 )</td>
<td>id( x = 2 )</td>
</tr>
<tr>
<td>-0.05</td>
<td>14390</td>
<td>0</td>
<td>9245</td>
</tr>
<tr>
<td>-0.03</td>
<td>13864</td>
<td>0</td>
<td>8446</td>
</tr>
<tr>
<td>0</td>
<td>13577</td>
<td>0</td>
<td>7568</td>
</tr>
<tr>
<td>TEST A3</td>
<td>( dx )</td>
<td>id( x = 1 )</td>
<td>id( x = 2 )</td>
</tr>
<tr>
<td>-0.05</td>
<td>10686</td>
<td>6258</td>
<td>16128</td>
</tr>
<tr>
<td>-0.03</td>
<td>9547</td>
<td>6930</td>
<td>15644</td>
</tr>
<tr>
<td>0</td>
<td>8107</td>
<td>8107</td>
<td>15371</td>
</tr>
<tr>
<td>TEST A4</td>
<td>( dx )</td>
<td>id( x = 1 )</td>
<td>id( x = 2 )</td>
</tr>
<tr>
<td>-0.05</td>
<td>7038</td>
<td>0</td>
<td>9013</td>
</tr>
<tr>
<td>-0.03</td>
<td>8056</td>
<td>0</td>
<td>10017</td>
</tr>
<tr>
<td>0</td>
<td>9845</td>
<td>0</td>
<td>11785</td>
</tr>
</tbody>
</table>
For a 4PCBB-1234 bearing, the algorithm convergence was tested according to the “Pseudo code” for 2 different cases assuming a contact radial load:

- for a High axial load

<table>
<thead>
<tr>
<th>Iter no.</th>
<th>Initial sol., $\beta_w$, deg</th>
<th>$Q_1$ [N]</th>
<th>$Q_2$ [N]</th>
<th>$Q_3$ [N]</th>
<th>$Q_4$ [N]</th>
<th>Computed value of, $\beta_w$, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>413.3</td>
<td>0</td>
<td>1422.4</td>
<td>990.3</td>
<td>-10.27</td>
</tr>
<tr>
<td>1</td>
<td>-10.27</td>
<td>406.1</td>
<td>0</td>
<td>1275.3</td>
<td>1143.4</td>
<td>-6.71</td>
</tr>
<tr>
<td>2</td>
<td>-6.71</td>
<td>408.1</td>
<td>0</td>
<td>1324.1</td>
<td>1088.7</td>
<td>-7.95</td>
</tr>
<tr>
<td>3</td>
<td>-7.95</td>
<td>406.6</td>
<td>0</td>
<td>1305.9</td>
<td>1107.8</td>
<td>-7.50</td>
</tr>
<tr>
<td>4</td>
<td>-7.50</td>
<td>407.3</td>
<td>0</td>
<td>1312.6</td>
<td>1100.9</td>
<td>-7.67</td>
</tr>
<tr>
<td>5</td>
<td>-7.67</td>
<td>406.9</td>
<td>0</td>
<td>1310.0</td>
<td>1103.4</td>
<td>-7.60</td>
</tr>
</tbody>
</table>

- for Low axial load

<table>
<thead>
<tr>
<th>Iter no.</th>
<th>Initial sol. $\beta_w$, deg</th>
<th>$Q_1$ [N]</th>
<th>$Q_2$ [N]</th>
<th>$Q_3$ [N]</th>
<th>$Q_4$ [N]</th>
<th>Computed value of, $\beta_w$, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>403.82</td>
<td>314.8</td>
<td>1405.9</td>
<td>1318.5</td>
<td>-1.71</td>
</tr>
<tr>
<td>1</td>
<td>-1.71</td>
<td>407.89</td>
<td>309.5</td>
<td>1389.9</td>
<td>1333.5</td>
<td>-1.50</td>
</tr>
<tr>
<td>2</td>
<td>-1.50</td>
<td>407.68</td>
<td>310.0</td>
<td>1392.1</td>
<td>1331.5</td>
<td>-1.54</td>
</tr>
<tr>
<td>3</td>
<td>-1.54</td>
<td>407.69</td>
<td>309.9</td>
<td>1391.7</td>
<td>1331.9</td>
<td>-1.53</td>
</tr>
<tr>
<td>4</td>
<td>-1.53</td>
<td>407.69</td>
<td>309.9</td>
<td>1391.8</td>
<td>1331.8</td>
<td>-1.53</td>
</tr>
</tbody>
</table>

CONCLUSIONS

The numerical results confirm the mixed control criteria presented by Nelias [3]. To evaluate the complex kinematics in the 2, 3 or 4 point contact ball bearing a computer code was developed. It implements a unitary mathematical model with 5DOF able to describe the quasi-static and quasi-dynamic behavior of lubricated ball bearings [6].

REFERENCES