OPTIMISATION OF THE TOOTH FLANK GEOMETRY FOR NON-STANDARD SPUR GEAR

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ABSTRACT
In order to increase the transmissible power level of the plastic spur gears, a non-standard tooth is designed, with a variable geometry along the gear facewidth. The variation of the tooth flank geometry, generated by a simple kinematics, is a consequence of a specific cutting-tool geometry and position. Due to the gear tooth complex geometry, the traditional numerical modeling and analysis of the 3D surfaces is replaced by the solid modeling technique. The non-standard gear generation process is simulated and the virtual gears are further used to investigate the gear basis formats and other criteria of gear quality. The data base unable the selection of the tooth flank geometry that would lead to a proper gear mesh.

KEYWORDS: Non-standard spur gear, plastic gear, gear basis formats, gear mesh, solid modeling.

1. INTRODUCTION
The metal curved facewidth spur gears are not very popular for the gear industry due to their limitations:
- a variable tooth geometry along the gear facewidth, depending on the cutting tool geometry and the kinematics of the gear generation process [1];
- the difficulty of mounting in a gear train and, for certain tooth geometry, the sensitivity to center distance variation;
- there is no design standard;
- not so much has been published on gear performances.

There is no doubt the curved facewidth spur gears have advantages on their spur counterparts, such as:
- higher contact ratio;
- higher tooth stiffness;
- smoother running due to their gradual mesh;
- better meshing in plane misalignment conditions.

On the other hand, the gear industry pays great interest in polymer and composite gears, trying to enlarge their utility; there is a continuous process of improving plastic gear performances, based on both modifications in traditional tooth gear geometry [2] and developments of new composite materials. This will enable plastic gear to be used in power transmissions.

Considering the curved facewidth spur gear advantages and trying to provide superior performances for plastic gears, the authors have especially designed a particular gear tooth, with variable geometry along the gear facewidth [3]. The double tooth curvature as well as the reduction in gear tooth height, away from gear center, will increase the tooth contact and bending resistance.

An analysis of the designed non-standard gear quality assumes that the basic gear format is identified and performance criteria are evaluated. Due to gear complex geometry, the traditional numerical analysis becomes extremely difficult [4] and is successfully replaced by solid modeling. The simulation of the gear generation process enables the virtual gears to be produced and manipulated in order to identify gear tooth geometry, gear mesh and gear resistance, by FEA [5].

2. GEAR TOOTH GEOMETRY
The specific tooth geometry is generated by simple gear generation process kinematics and simple tools geometry; two parameters control the generating process and define the tooth geometry: \( R_g \) – the radius of the “generating circle” and \( \beta \) – the tool axis inclination. Since the tooth geometry varies in sections along the gear facewidth, there are important questions about the limits the generating parameters are chosen, considering a further correct mesh. It is obvious that:
- the reduction in tooth height influences the gear backlash, one of the fundamental gear data;
- the curvature of the gear tooth, along the gear facewidth, influences the tooth thickness; second order interference could occur during the gear manufacture,
especially at low values of the generating circle radius.

It should be mentioned that the non-standard gear tooth geometry is of standard shape and dimensions at gear center section and varies along the gear facewidth; important modifications of tooth geometry are recorded at gear facewidth ends, therefore measurement of gear tooth parameters are developed especially on these gear sections.

Solid modeling is used to simulate the gear generation process, based on a simple kinematics:
- the tools perform rotational motions about their inclined axis; these motions are replaced by conical solids, designed for both concave and convex tooth flanks generation. Sections of the conical tools have variable dimensions and orientation, conforming to the generating parameters, \( R_g \) and \( \beta \) (fig. 1);
- the gear cylindrical blank performs related rotational and translational motion, tangential to the base circle, to provide the rolling motion required for the involute tooth form generation; a cylindrical solid is designed for the gear blank, conforming to the chosen defining parameters: gear modulus \( m \), number of teeth \( z \) and gear facewidth \( B \).

As a first step, the Tooth generation. lsp routine is concerned with one tooth “manufacture”, in order to investigates the fundamental gear data in sections along the gear face width. Figure 1 shows a schematic representation of the routine:

Figure 2 illustrates the generated tooth, for a gear with the following design parameters: \( m=2\text{mm}, z=30, B=24 \text{ mm}, R_g=40\text{mm}, \beta=10^\circ \).

**Fig. 2.** Representation of the curved facewidth spur gear, with variable tooth height.

2.1. Influence of generating parameters variations on gear geometry

To verify the range of both generating parameters - tooth curvature along the gear facewidth and tool axis inclination – variations, simulations of the gear generation process were developed. Two important errors were recorded (fig. 3):
- the second order interference occurs during the gear blank rolling motion, at low values of the radius \( R_g \);
- a negative backlash results at high values of the tool axis inclination \( \beta \), in sections away from gear center.

Table 1 presents the results of investigations on the generating parameters variations, considering a spur gear defined by \( m=2\text{mm}, z=30, B=24 \text{ mm} \).

2.2. Tooth backlash and thickness

The peculiarity of the designed non-standard gears consists in the variable tooth height along the gear facewidth, decreasing away the gear centre. As shown previously, the reduced tooth height influences the tooth backlash, an important criteria of the gear quality. Thus, there could be gear geometries that would request extended center distance, for a proper mesh.
Table 1. Tool axis inclination versus tooth curvature.

<table>
<thead>
<tr>
<th>$R_g$ [mm]</th>
<th>$\beta$ variation [°]</th>
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<tbody>
<tr>
<td>16</td>
<td>Interference [5 6] Negative backlash</td>
</tr>
<tr>
<td>17</td>
<td>Interference [2 6] Negative backlash</td>
</tr>
<tr>
<td>18</td>
<td>[0 7] Negative backlash</td>
</tr>
<tr>
<td>20</td>
<td>[0 8] Negative backlash</td>
</tr>
<tr>
<td>30</td>
<td>[0 12] Negative backlash</td>
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<tr>
<td>40</td>
<td>[0 16] Negative backlash</td>
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<tr>
<td>60</td>
<td>[0 22] Negative backlash</td>
</tr>
<tr>
<td>80</td>
<td>[0 33] Negative backlash</td>
</tr>
<tr>
<td>100</td>
<td>[0 43] Negative backlash</td>
</tr>
<tr>
<td>300</td>
<td>[0 88] Inconsistent tools</td>
</tr>
</tbody>
</table>

The Sections.lsp routine enables sections along the gear facewidth to be produced and defining geometrical parameters to be measured, such as: base circle, dedendum circle, tooth thickness. Figure 4 illustrates the tooth height variation for extreme values of tool axis inclination, in case of a tight curvature ($R_g=20mm$). While the tool axis inclination is increased from $2°$ to $7°$, the tooth backlash, related to its standard value, is reduced from 73.4% to 7.4% at the end of the gear facewidth.

$\beta = 2°$  $\beta = 7°$

Fig. 4. Tooth height variation for $R_g=20mm$.

Figure 5 illustrates the tooth height variation for extreme values of tool axis inclination, in case of a large curvature ($R_g=100mm$). The tool axis inclination could be increased to $43°$, as shown in table 1; for $\beta=40°$, the tooth backlash is reduced from the standard value of 0.5 mm to 0.035 mm (7%) at the end of the gear facewidth.

A synthesis of the tooth backlash variation, related to both generating parameters, is presented in figures 6-9. Measurements are specific to the gear facewidth ends.

The analysis of the tooth backlash variations shows new limitations of the generating parameters. In order to avoid interference, the backlash should be higher than the standard value $c=0.1m$; i.e., for 2 mm modulus, the tooth backlash at the dedendum requires a generation radius higher than 23 mm. Also, the increase in tool axis inclination should be limited at $7.6°$.

Figures 8 and 9 illustrate the variation of the tooth thickness, in sections at the gear facewidth end, related to the standard value $s_{a0}$. It can be seen that, for the lowest admissible value of the generating radius ($R_g=23mm$ – fig. 8), the tooth addendum is about 6.8% higher than the standard value; the simulation of the gear mesh showed that no interference occurred for this modified tooth geometry. The variation in tool axis inclination induces an increase of 4.7% in tooth addendum thickness and is slightly reduced as the $\beta$ parameter is increased. The tooth thickness variation shows that there is no danger in tooth sharpening.
3. GEAR MESH

A simulation of the gear mesh is developed in order to study the influence of the tooth modified geometry on the path of contact. There are two steps considered for the gear mesh analysis:

- The Gear Positioning.lsp routine enables to identify the geometrical parameters of the gears contact, such as length of the line of contact, angular positions of the extreme points where gear come into and out of mesh, in sections away from the gear center. Considering the above calculated parameters, the pinion and gear are rotated by so the teeth mesh starts with contact in the gear center section, with standard geometry (fig. 10);
- The Gear Mesh.lsp routine enables the evolution of the path of contact to be illustrated during one tooth pair complete mesh.

Figures 10 to 12 show paths of contact for non-standard gears with tooth geometry defined in paragraph 2; gear ratio \( i=1 \). The parameters \( \beta \) and \( R_g \) are chosen at some extreme admissible values, as mentioned in the figure titles.
Fig. 13. Path of contact at the beginning of mesh (a) and during the gear mesh at 4°(b), 8°(c), 16°(d) and 22°(e) pinion rotational angle; Rg=100 mm, β=30°.

Analyzing the path of contact for gears with different tooth curvature, at the same tool axis inclination, it was noticed that:
- the area at the tooth contact is significantly reduced for high tooth curvature. Considering a teeth interference of 0.1 mm, at 4° pinion rotational angle, the path surface decreases from 27.24 mm² (Rg=100mm) to 15.96 mm² (Rg=30mm);
- a longer tooth contact occurs in case of gear tooth with high curvature, so the gear contact ratio is increased. As is shown in fig. 12, at some stage, there are three pairs in contact.

Figures 12 and 13 enables a comparison between paths of contact in case of a gear with the same tooth curvature, but different tooth height variations. The tooth contact area increases slightly from 27.24 mm² (β=5°) to 29.12 mm² (β=30°); close to the position a tooth pair comes out of mesh, it appears that another two pairs are in contact.

4. CONCLUSIONS

A non-standard geometry is designed for the plastic spur gears in order to increase the transmissible power level. The double curvature of the gear tooth depends on two parameters: the tool axis inclination and the radius of the generating circle.

The gear geometry variation is limited by the standard specifications on criteria of gear quality. The simulation of the gear generation process, using solid modeling, enabled tooth geometry to be investigated and limitations in generating parameters variations to be produced.

A simulation of the gears mesh was developed to produce the path of contact. Several illustrations showed the path of contact shape, area and distribution along the gear teeth. It was shown that the decrease in the generating radius as well as the increase in tool axis inclination led to a higher gear contact ratio.

REFERENCES