ANALYSIS OF GEAR QUALITY CRITERIA AND PERFORMANCE OF CURVED FACE WIDTH SPUR GEARS

Laurentia ANDREI¹, Gabriel ANDREI², Douglas WALTON², Karl DEARN²

¹University “Dunarea de Jos” of Galati, ROMANIA, ²University of Birmingham, UK
laurencia.andrei@ugal.ro

ABSTRACT

For plastic gears, developments in composites and tooth geometry are essential for their selection in power transmissions. In order to enhance the transmissible power level of the plastic spur gears, an arc-shaped tooth is designed, with variable height along the gear face width. There are two parameters that influence the tooth geometry: the radius of the generating point, defining the tooth curvature, and the tool axis inclination, leading to a tooth height variation. The paper deals with computerized design of the curved face width spur gear, as a replacement of the traditional numerical procedure, focused on tooth surface generation, contact analysis and stress calculation. The procedure enabled the correlation of the gear defining parameters such that interference could be avoided. The gears mesh simulation illustrated the path of contact geometry and enabled a proper load distribution for the finite element analysis. Compared to other standard spur gears, the designed curved face width spur gears exhibited a higher bending resistance.

Keywords: Non-standard gear, curved face width tooth, plastic gear, gear mesh

INTRODUCTION

Demands on the power density of plastic gears necessitate the use of performance composites as well as modifications to the tooth geometry [1, 2, 3]. The curved face width spur (CFWS) gear, examined in this paper, is a non-standard gear, with a spatial tooth system (arc-shaped) and a variable involute for the tooth flank topography. CFWS gears are not very popular in the gear industry, despite providing a high contact ratio, high tooth stiffness, smoother running, lower contact and bending stresses, improved mesh when misaligned and no axial forces [4]. However, CFWS gears are difficult to mount in a gear train and, for certain geometries, they are sensitive to centre distance variation. Especially designed for plastic gearing, in order to increase their transmissible power levels, the CFWS gears have a specific geometry, defined by two generating parameters:

- the radius of the generating point (Rg) which influences the tooth curvature along the gear face width, and
- the tool axis inclination (β) influencing the tooth height variation along the gear face width.

Due to the complex tooth geometry, the numerical simulation of the gear generation and mesh [5] is replaced by a solid modelling method. In this paper, a software package, developed in AutoCAD - AutoLISP, is used for:

- the simulation of the gear generation process.
This simulation is applied to some practical problems, e.g. measurements on the virtual gear, in order to establish the optimum tool geometry and position, analysis of the influence the modified tool parameters have on the tooth flank topography;
- the simulation of the gear meshing cycle enabling the analysis of the path of contact, both in curved face width spur gears and standard spur gears.
GEAR TOOTH GEOMETRY

The primary manufacturing method of plastic gears is injection molding, which enable almost any tooth profile to be generated. However, due to the complex tooth geometry of the CFWS gear, the design of a molding die is almost impossible. Therefore, the curved face width spur gears are machined by means of a bespoke machine tool [6].

As the gear tooth surface is a function of the cutter design parameters, an optimization process of the tool geometry and position is first required. This must consider the gear backlash and clearance as standard criteria of gear quality. The optimisation process of the gear tooth geometry is based on the following procedure (fig. 1):

Step 1 - The gear generation process start up refers to the cutting tool position, defined by both $R_g$ and $\beta$.

Step 2 – The cutting process kinematics consists of the tool rotational motion about its inclined axis (I) and the gear blank rotational motion (II) related to the translational motion (III)) in order to assure the rolling motion required for the involute flank generation.

During the cutting process, tooth generation error could occur, such as primary interference or sharpening of the gear tooth, at the end of the gear face width. This will eventually affect the involute flank. Therefore, the lowest values of the defining parameter - the radius of the generating circle, related to the gear face width $B$, must be evaluated to avoid generation errors during the rolling motion.

Figure 2 shows the variation of the minimum allowed generating radius versus the gear face width, for a zero tool axis inclination; the gear geometry is defined by a modulus of 2 mm and 30 teeth. A non-dimensional parameter, $K$, is used to express the allowable tooth curvature along the gear face width, given as:

$$K = \frac{R_{g\text{min}}}{B}$$

Fig. 1. A schematic illustration of the gear tooth geometry optimisation process

A finite element method is then applied to determine the influence of the tooth flank modifications on gear bending strengths.
Simulation of the gear generation process shows that:
- for values of the gear face width up to 28 mm, the generating radius is conditioned by the primary interference that occurs during the rolling motion;
- for \( B > 28 \) mm, low values of the generating radius lead to a concave non-involute tooth flank away from the gear centre.

**Step 3** – The tooth geometry investigation process is focused on measurements of the tooth thickness and tooth height, in sections at the end of the gear face width, where the tooth geometry modifications could affect the gear operating conditions. Due to the kinematics of the generating process, the tooth flank profile is near an involute, in sections along the gear face width, where each profile is defined by a slightly different base circle \((R_{bc})\). The domain of the variation in the parameters is limited by maximum measured values of the measured tooth thickness and height that would lead to negative backlash and/or clearance. Table 1 presents the variation of the tool axis inclination for a gear geometry defined by a 2 mm modulus, 30 teeth and gear face width of 24 mm. Several tooth curvatures were tested.

### Table 1. Limitation of the tool axis inclination.

<table>
<thead>
<tr>
<th>( K )</th>
<th>( \beta ) variation ([\degree])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.83</td>
<td>[0 \ldots 8] - Negative clearance</td>
</tr>
<tr>
<td>1.25</td>
<td>[0 \ldots 12] - Negative clearance</td>
</tr>
<tr>
<td>1.66</td>
<td>[0 \ldots 16] - Negative clearance</td>
</tr>
<tr>
<td>2.5</td>
<td>[0 \ldots 22] - Negative clearance</td>
</tr>
<tr>
<td>3.33</td>
<td>[0 \ldots 33] - Negative clearance</td>
</tr>
<tr>
<td>4.16</td>
<td>[0 \ldots 43] - Negative clearance</td>
</tr>
<tr>
<td>12.5</td>
<td>[0 \ldots 88] - Inconsistent tools</td>
</tr>
</tbody>
</table>

**TOOTH BENDING STRENGTH**

The COSMOS/M program has been used to evaluate mainly the CFWS gear bending behaviour. The finite element analysis is performed for non-standard and standard gear geometry in order to determine:
- the benefit/limitation the curved face width spur gears have over their counterparts, and
- the influence the modified tooth geometry has on gear bending strength.

A small sector of the gear, consisting of 3 teeth, is modelled by 8-noded SOLID elements (table 2), allowing just translational motions. For the tooth strength analysis, the tensile modulus of elasticity of the gear plastic material was 3450 MPa and Poisson’s ratio was 0.3. The teeth were statically loaded with a torque of 4 N/mm\(^2\), in accordance with the configuration of the path of contact.

### Table 2. Gear sector models.

<table>
<thead>
<tr>
<th>Gear geometry</th>
<th>Elements</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spur geometry</td>
<td>62855</td>
<td>67179</td>
</tr>
<tr>
<td>Herringbones gear</td>
<td>61101</td>
<td>65415</td>
</tr>
<tr>
<td>CFWS gear ( K = 1; \beta = 0\degree )</td>
<td>62740</td>
<td>67179</td>
</tr>
<tr>
<td>CFWS gear ( K = 2; \beta = 0\degree )</td>
<td>62839</td>
<td>67277</td>
</tr>
<tr>
<td>CFWS gear ( K = 2; \beta = 5\degree )</td>
<td>60696</td>
<td>65121</td>
</tr>
<tr>
<td>CFWS gear ( K = 2; \beta = 10\degree )</td>
<td>60204</td>
<td>64631</td>
</tr>
<tr>
<td>CFWS gear ( K = 3; \beta = 0\degree )</td>
<td>62550</td>
<td>66983</td>
</tr>
</tbody>
</table>

The finite element analysis shows that:
- for the CFWS gears, the maximum stresses are recorded towards gear centre sections;
- the von Mises stresses distribution is relatively uniform along the gear face width in both the standard and CFWS gears;
- there is a strong interference between the stress distributions on the loaded and unloaded tooth flanks of standard spur gears compared to others;
- an increase of 15…20% of the maximum Von Mises stresses is exhibited when the load is applied on the convex tooth flanks of the CFWS gears.

Figure 3 shows a comparison of the bending stresses and deflections between a spur gear and a CFWS gear, with large tooth curvature along the gear face width \((K=3)\). When two teeth pairs are in contact, the maximum Von Misses stresses in the CFWS gear, predicted at 2.58 N/mm\(^2\), are reduced by 11% compared to the standard spur gear. The maximum recorded deflection for the CFWS gears is 0.098 mm for double tooth pair contact; this is compared to 0.01 mm for the standard spur gear.

Figure 4 compares the analysed bending stresses and tooth deflections between a herringbone gear and a curved face width gear, defined by \( K=1 \). The helix angle of the herringbone gear is specified such that the tooth is tangent to the CFWS gear tooth, with minimum length related to the gear face width. It was found that the Von Misses stresses, calculated in the herringbone gear centre section, at the tooth dedendum, are 2.318 N/mm\(^2\) on the loaded flank, slightly increasing to the maximum value of 2.48 N/mm\(^2\) on the opposite flank. For the CFWS gear, the maximum Von Misses stresses is found at 2.23 N/mm\(^2\), 10% lower than the stresses specific to the previous geometry. As regards the tooth deflections, the maximum values are recorded as 10.3 \( \mu \)m and 9.67 \( \mu \)m for the herringbone and CFWS gears respectively.

In the previous analysis, the arc-shaped tooth had a constant height along the gear face width. The next step is to consider both variations of the tooth curvature and height, and to analyse the influence of the tooth geometry on the gear bending resistance.
Figure 3. Showing the path of contact, the von Mises stresses and deflections at 4 Nm torque, for standard spur gear (a) and CFWS gear defined by $R_g=72$ mm, $\beta = 0^\circ$.

Figure 4. Showing the path of contact, the von Mises stresses and deflections at 4 Nm torque, for herringbone gear (a) and CFWS gear defined by $R_g=24$ mm, $\beta = 0^\circ$.

Figure 5a illustrates the tooth bending stresses for similar regions of the curved face width spur gears with tooth geometry defined by different $R_g/B$ ratios ($K$), in case of a zero tool axis inclination. It can be seen that:
- for a certain load, the von Mises stresses increase as the generating radius is increased;
- the higher the applied torque, the less significant the increase in tooth bending stresses. As the torque is increased by 200%, the maximum stress variation is reduced by 81%.

Figure 5b lists the results of the tooth bending stresses and deflections for the curved face width spur gears with tooth geometry defined by $R_g/B=2$, at different tool axis inclinations. It can be seen that the tooth height variation along the gear face width does not significantly influence the tooth bending resistance, but generally increases the von Mises stresses.
As regards the influence of gear generating parameters on gear tooth stiffness, it was noticed that:
- a high curvature will reduce the maximum tooth deflections, while
- an increase of the tool axis inclination will slightly increase the tooth deflections.

Table 3 presents the maximum recorded values of the tooth deflections, specific to the curved face width spur gear, designed by the standard geometric parameters mentioned above.

Table 3. Influence of the non-standard gear geometry on the tooth stiffness.

<table>
<thead>
<tr>
<th>Tooth geometry</th>
<th>Maximum deflection [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K=1$, $\beta=0^\circ$</td>
<td>0.00967</td>
</tr>
<tr>
<td>$K=3$, $\beta=0^\circ$</td>
<td>0.0101</td>
</tr>
<tr>
<td>$K=2$, $\beta=0^\circ$</td>
<td>0.00994</td>
</tr>
<tr>
<td>$K=2$, $\beta=10^\circ$</td>
<td>0.0102</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

The curved face width spur gear, with modified geometry, is designed for plastic gears in order to enhance their transmissible power level, using a tooth double curvature along the gear face width and a variable height.

The solid modelling procedure was used for virtual gear generation and mesh. The gear generation process simulation enabled investigations on the complex tooth profile geometry and a correlation of the defining parameters such that:
- the tooth curvature versus the gear face width would avoid primary interference, and
- the tool axis inclination versus the gear face width would avoid secondary interference.

The gear mesh simulation illustrated the path of contact geometry in both the standard and curved face width spur gears. On this basis, loads were properly distributed for the finite element analysis. It was found that the curved face width spur gear tooth curvature, as well as the reduced tooth height, increased the gear bending resistance as compared with the spur and herringbone gears.

**REFERENCES**