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**SOME ASPECTS ABOUT DRIVING WHEEL/RAIL CONTACT IN STEADY STATE INTERACTION**

Traian MAZILU

POLITEHNICA University of Bucharest, Dept. of Railway Vehicles, ROMANIA

trmazilu@yahoo.com

**ABSTRACT**

The driving wheel/rail steady state interaction and the response to harmonic excitation were studied using a complex model for track. The starting point for numerical simulations is the Green matrix of the track. The wheel/rail displacements and the contact forces have been calculated having the aim of analysing the particularly case of driving wheel/rail interaction which is very interesting for corrugation wear issue.

The driving wheel/rail interaction behaviour due to parametric excitation caused by the discretely support of the track is dominated by the first own frequency for wheel/rail system. When the passing frequency over the sleepers equals the first own resonance frequency, the wheel/rail contact forces (the normal force and the traction force) have the highest value and the wheel rolls over the rail at the critical speed. The numerical results show that the effective traction force can be 8.5% from the mean traction force. Consequently, the railway traffic at the critical speed must be avoided.

Generally, the driving wheel/rail interaction is characterised by the superposition of the roughness and parametric excitation due to sleepers. The modulated oscillation phenomenon occurs – the carrier is the effect of the excitation due to roughness and the modulated is given by the parametric excitation. The spectral components of both the carrier and the modulated can be added in the pinned-pinned resonant/anti-resonant frequency range, especially. Due to this effect, the effective traction force is very sensitive to the roughness wavelength of the running surface and contributes to the rail corrugation.

The effective traction force increases as the tractive power increases. This aspect explains why the heavy locomotives are more aggressive for rail.

**Keywords:** driving wheel, rail, steady state interaction.

**INTRODUCTION**

When a loaded smooth wheel rolls without slip on a smooth rail supported by sleepers, wheel/rail interaction is caused only by the parametric excitation of the track due to the varying dynamic stiffness of the discretely support. It is the ideal case of wheel/rail interaction, when the dynamic force is minimal. This interaction behaviour is known as the so-called ‘wheel/rail steady state interaction’ or ‘wheel/rail interaction due to parametric excitation’.

The dynamic force of wheel/rail contact proper to the wheel/rail steady state interaction, is periodical and its own fundamental frequency equals the ratio between the speed and the span length. The frequencies of the other spectral components are multiples of the fundamental frequency.

When the real wheel rolls along a rail, both are set in vertical vibration by the irregularities (roughness, waviness) and the parametric excitation due to discretely support of the track. The result is an amplitude modulated vibration, in which the carrier frequency is caused by roughness excitation and the modulated is given by the steady state interaction. Nordborg [1], Hou et al. [2], Wu and Thompson [3], Mazilu [4] and others studied the wheel/rail steady state interaction.
The study of vibration generated by a rolling wheel on a rail is critical in predicting the short-pitch rail corrugation and wheel/rail noise. Short pitch corrugation shows as an undulation of the rail surface with typical wavelength between 25-80 mm. On the surface of severely corrugated rails the amplitudes can reach values of 50…100 µm. Several different damage mechanisms have been suggested to be the cause of short-pitch rail corrugation [5, 6]. The wheel/rail wear produces especially during the driving or braking behaviour. Many papers have been dedicated to predict the growth of railhead roughness [7, 8, 9]. The starting point of these studies is numerical simulation of dynamic train-track interaction. The main hypothesis is that wear is caused by longitudinal slip due to driving wheel. Further, the wear is proportional to the longitudinal power in the contact patch. At the driving wheel/rail contact, the normal force and the tangential force (creep force) act on the two elastic bodies. The issue of contact forces is resolved by Kalker [10], but his theory is difficult to apply in numerical simulations.

Many other simplified solutions were proposed such as Polach’s method [11, 12].

The short-pitch wavelength is related to the high frequency of the traction force due to wheel/rail normal force. The wheel/rail steady state interaction in the presence of the rail roughness is the cause of the wheel/rail normal force history. Present paper deals with the dynamic response of driving wheel to the varying normal force due to steady state interaction. To the author’s knowledge, this particularly issue has not been studied in the past.

2. MECHANICAL MODEL

The mechanical model has three parts: the track, the driving wheel and wheel/rail contact as depicted in figure 1. The track is reduced to a rail supported by rail-pads, semi-sleeper and ballast. The rail is taken as a uniform infinite Timoshenko beam. For longitudinal dynamics, the rail is treated as an infinite simple bar. The parameters for the rail are: the mass per length unit $m$, the Young’s modulus $E$, the shear modulus $G$, the density $\rho$, the cross-section area $S$, the area moment of inertia $I$ and the shear coefficient $k$. The distance from the rail foot to the cross-section neutral fibre is $h$. The loss factor of the rail is neglected. The dynamics of the rail are: the vertical displacement $v(x, t)$, the rotation of the cross-section $\theta(x, t)$ and the longitudinal displacement $u(x, t)$.

The vertical and longitudinal dynamics of the rail is coupled to the rail pads. The rail pad is modelled as a parallel connection of spring and dashpots with linear characteristics in vertical and longitudinal (along the rail) dimensions and a similar torsion spring and damper restraining rotation in vertical plan. The elastic constants are $k_{ss}$, $k_z$ and $k_{zz}$ and the viscous damping constants are $c_{ss}$, $c_z$ and $c_{zz}$.

The ballast is represented by a system of springs and dashpots having viscous damping in vertical and longitudinal directions. The elastic constants $k_{bs}$, $k_{bz}$ and the viscous damping constants $c_{bs}$, $c_{bz}$ are related to the ballast.

The wheel is regarded as a loaded disc which rolls and slides on the rail at constant $V$ speed and variable $\omega$ angular speed. The parameters for the wheel are: the mass $M_w$, the nominal rolling radius $r_w$ and the dead load $P_o$. A torque $T$ acts on the wheel and his value depends on the speed so that the tractive power to be constant.

The equations of motion are
- for the track
  \[
  L_{xx} \{q\}_i + \sum_{i=1}^{i=z} (A_i \{q\}_i + B_i \{q\}_j) \delta(x-x_i) = \{p\}_i
  \]  
  \[
  C_i \{q\}_i = D_i \{q\}_i
  \]
  \[
  \text{where } L_{xx}, A_i, B_i, C_i \text{ and } D_i \text{ stand for matrix differentials, } \{q\}_i = \{q(x, t)\} = \{u(x, t) \quad v(x, t) \quad \theta(x, t)\}^T \text{ is the column vector of rail displacements, } \{q\}_s = \{q(s, t)\}, \]
  \[
  \{q\}_s = \{q_i(s, t)\} = \{x_i(t) z_i(t) \alpha_i(t)\}\]
  \[
  \{p\}_s = -P(t) \delta(x-Vt)\{e\}_s \text{ is the column vector of forces on the rail where } P(t) \text{ is the wheel/rail normal contact force, } \{e\}_s = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \text{ and } \delta(x) \text{ is Dirac’s delta function.}
  \]
- for the wheel
  \[
  M_w \ddot{\theta}_w(t) = P_o - P(t)
  \]  
  \[
  I_w \dot{\omega}_w(t) = T - r_w F(t)
  \]  
  \[
  R(t) = F(t)
  \]
  \[
  \text{where } F(t) \text{ is the traction force and } R(t) \text{ is the resistance force.}
  \]
  \[
  \text{After Polach } [11, 12], \text{ the traction force is}
  \]
  \[
  F = -2P\frac{\mu}{\pi} \left( \frac{e}{1 + \frac{e}{\varepsilon}} \right)\]  

![Fig. 1. Mechanical model for the driving wheel/rail steady state interaction: 1 – rail, 2 – rail-pad, 3 – semi-sleeper, 4 – ballast, 5 – driving wheel, 6 – stiffness contact.](image)
with
\[
\varepsilon = \frac{2 \pi C a^2 b}{3 \mu P} s
\]
where \( \mu \) is the coefficient of friction, \( \varepsilon \) is the gradient of the tangential stress in the area of adhesion, \( C \) is proportionally coefficient characterising the contact shear stiffness, \( a \) and \( b \) are the semi-axes of the contact ellipse and \( s \) is the creep. For simplicity, the argument \( t \) has been omitted. The semi-axes of the contact ellipse are calculated according to the theory of Hertz.

If the contact shear stiffness coefficient is derived from Kalker’s theory (linear) \([10]\), then the gradient \( \varepsilon \) is given by
\[
\varepsilon = \frac{\pi abc_i_j G}{4 P} s
\]
where \( G \) is the shear modulus and \( c_{ij} \) is coefficient from Kalker’s linear theory.

For traction behaviour, the creep has the formulae \([13]\)
\[
s = \frac{V - r_n \omega}{r_n \omega} = \frac{V}{V}
\]
The wheel/rail normal force is expressed by
\[
[P(t) / C_n]^{1/3} = z_n(t) H[z_n(t)]
\]
where
\[
z_n(t) = z_n(t) - v(Vn, t)
\]
is wheel/rail deflection, \( C_n \) represents the Hertzian constant and \( H[\_] \) is the Heaviside function.

The boundary conditions are
\[
limit_{t \rightarrow \infty} [q(x,t)] = [0 0 0]^T,
\]
\[
limit_{t \rightarrow \infty} [q^x \ (x,t)] = [0 0 0]^T
\]
and all initial conditions are set null.

The differential equations of motion may be solved through numerical integration following two steps.

Firstly, the equations (1-3) and (10-11) are solved using an original Green functions method \([15]\).

For this aim, the Green function’s column vector for rail displacement
\[
\{g(x, \xi, t - \tau) = \{g_n(x, \xi, t - \tau), g^x(x, \xi, t - \tau)^T\}
\]
was calculated starting from the complex Green functions of the rail (the receptances).

To this end, the inverse Fourier transform is applied in numerical approach. Then, the rail displacement at the contact point may be determined
\[
v(Vn, t) = \int_{-\infty}^{0} \int_0^\infty g^*(Vn, \xi, t - \tau) P(\tau) \delta(\xi - V\tau)d\tau d\xi = \int_0^\infty g^*(Vn, V\tau, t - \tau) P(\tau)d\tau
\]
The wheel vertical displacement and speed are
\[
z_n(t) = z_n(0) + \frac{1}{M_n} \int_0^t [P_0 - P(\tau)]d\tau
\]
Practically, a certain \( t_0, t_1, \ldots, t_n \) (with \( t_0 = 0, t_n = t \)
and \( \Delta t = t_i - t_{i-1} \), where \( i = 1, \ldots, n \)) must be considered. The rail displacement at the contact point and the wheel displacement and speed can be calculated with the formulae
\[
v(Vn, t_n) = \sum_{i=1}^{n} \int_0^t \left[g^*(Vn, V\tau, t_i - \tau) P(\tau) d\tau \right]
\]
\[
z_n(t_n) = z_n(t_{n-1}) + \int_0^t \dot{z}_n(\tau) d\tau
\]
\[
\dot{z}_n(t_n) = \dot{z}_n(t_{n-1}) + \frac{1}{M_n} \int_0^t \left[P_0 - P(\tau)\right]d\tau
\]
It is assumed that the contact force \( P(\tau) \) and the Green function will have a linear variation in the \( [t_{n-1}, t_n] \) time interval. We have
\[
\dot{z}_n(t_n) = \dot{z}_n(t_{n-1}) + \frac{\Delta t}{M_n} \left[rac{P_0 - P(t_{n-1})}{2}\right] + \frac{\Delta t^2}{2M_n} \left[rac{P_0 - 2P(t_{n-1})}{3}\right]
\]
Using the properties of the Green functions – they are attenuated in space, time and frequency – do- mains and they are periodic – \( g^*(Vn, V\tau, t_n - \tau) \)
can be organized as a matrix – the Green matrix of the track. For any time numerical simulation, the Green matrix of the track can be used. Finally, wheel/rail vertical displacements and normal contact force result.

The second step: the equations (4) and (5) are solved using the equations (6, 9) and normal force \( P(t) \). In fact, the angular speed is calculated with
\[
\omega(t_n) = \omega(t_{n-1}) + \frac{\Delta t}{I_n} \left[-r_n F(t_{n-1})\right]
\]
A short mention about equation (8) is necessary. The issue of the creep force – the traction force in this case – has two aspects. The first is the so-called ‘steady creep’, in which the normal force and the creep are constant.

When the loaded wheel rolls on the rail, the wheel/rail normal force is variable due to the parametric excitation. Further, the creep is variable too due to the torque of the wheel. In this case, we have the ‘unsteady creep behaviour’. In this paper, a quasi-static method is used to solve this unsteady rolling contact problem. It is assumed that in each

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tiny interval of time the wheel/rail contact is in steady state and Polačch’s formulae are applied. In fact, the semi-axes are calculated step by step.

3. NUMERICAL SIMULATION

Numerical simulations are carried out using the model introduced in the previous section for the driving wheel/rail steady state interaction. The values for the wheel parameters are: \( M_w = 750 \text{ kg}, \quad I_w = 68 \text{ kgm}^2 \) and \( r_w = 0.5 \text{ m} \). The static load \( P_0 = 100 \text{ kN} \) was considered.

The values for the track parameters are: \( m = 56 \text{ kg/m}, \quad I = 23.14 \times 10^{-6} \text{ m}^4, \quad S = 7.134 \times 10^{-3} \text{ m}^2, \quad E = 210 \text{ GPa, } \quad G = 85 \text{ GPa, } \quad \kappa = 0.34, \quad M_s = 129 \text{ kg, } \quad I_s = 0.82 \text{ kgm}^2, \quad d = 0.698 \text{ m, } \quad h = 0.08 \text{ m, } \quad h_0 = 0.085 \text{ m, } \quad h_1 = 0.089 \text{ m, } \quad k_{rx} = 34 \text{ MN/m, } \quad k_{rz} = 280 \text{ MN/m, } \quad k_{r\alpha} = 114.3 \text{ kNm, } \quad c_{rx} = 63 \text{ kNs/m, } \quad c_{rz} = 25.7 \text{ Nms, } \quad k_{bx} = 35 \text{ MN/m, } \quad k_{bz} = 180 \text{ MN/m, } \quad c_{bx} = 52 \text{ kNs/m and } \quad c_{bz} = 82 \text{ kNs/m.}

The Hertzian constant \( C_H = 9.36 \times 10^{10} \text{ N/m}^{3/2} \) was calculated for a wheel having a 1000 mm diameter and a conic rolling profile on UIC 56 rail.

The effective contact forces versus wheel speed are presented in figure 5.

![Fig. 2. Wheel/rail steady state interaction at speed of 48 m/s: a) wheel displacement; - - - - - - rail displacement at the contact point; sleeper position; b) angular speed.](image)

![Fig. 3. Normal contact force at 48 m/s; a) time-domain, sleeper position, b) spectrum.](image)

![Fig. 4. Traction force at 48 m/s; a) time-domain, sleeper position, b) spectrum.](image)

![Fig. 5. Effective contact force versus wheel speed: a) effective normal force; b) effective traction force.](image)
In fact, only the effective traction force is involved in corrugation wear. The effective traction force and the effective normal force have the same trend. The effective forces reach their maximum value at the critical speed (48 m/s). On the other hand, another local maximum value is detected when the wheel speed is about half the critical speed (24 m/s) because the second component spectral is highest. It has the frequency equal to the frequency of passing over the sleepers.

Figure 6 shows the influence of power at wheel for critical speed of the driving wheel/rail steady state interaction. The effective traction increases as the power at wheel increases. This increase is somewhat nonlinear.

Figures 7 and 8 display the contact force due to harmonic excitation by the wavelength of 93.33 mm, when the wheel rolls on the corrugated rail at speed of 60 m/s and its tractive power has the value of 200 kW. For numerical simulation, the term of the rail roughness has been added in the equation (11). The contact forces are modulated as it can be seen. In fact, the wheel/rail response due to steady state interaction is overlapped by the wheel/rail response due to roughness. In this case, the carrier frequency is caused by the sinusoidal roughness excitation and the modulated one is given by the interaction due to parametric excitation. The carrier has 643 Hz and the fundamental component of the modulated has 86 Hz. The spectrum has two distinctive components: the harmonics of the modulated and the modulated spectral components with the frequencies of \( f_c \pm k f_m \) where \( f_c \) stands for the frequency of the carrier, \( f_m \) stands for the frequency of the modulated and \( k \) stands for an integer number. The effective normal force has the value of 14.8 kN and the effective traction force has the value of 264 N.

When the carrier frequency is a multiple of the fundamental component of the interaction due to steady state interaction, both kind of components are overlapped. Figures 9 and 10 illustrate this aspect, the contact forces due to roughness with the wavelength of 70 mm and the same amplitude and wheel speed are presented. The effective forces have the value of 26.3 kN for the normal force and 545 N for the traction force.

Comparing the results from the two numerical simulations, the driving wheel/rail response is strongly influenced by the roughness wavelength.
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