A NUMERICAL SOLVER FOR THE WHEEL-RAIL CONTACT SUBJECT TO COMBINED LOADS

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ABSTRACT

A working algorithm, able to be incorporated into a computer code, has been developed to solve the stress state in the general case of non-Hertzian contacts. Brent’s method has been involved to find the contact point for the unload conditions. To limit the pressure, an elastic-perfect plastic material has been incorporated into the computer code. Purely normal loadings as well as normal and tangential Coulombian forces have been considered.

The elastic-plastic analysis model allows for a fast analysis of the influence of different parameters such as load level, contact geometry including the geometry of the worn profiles. It can be used to build the shakedown map for the material law of the rail material.

KEYWORDS: Rail-wheel contacts, elastic-perfect plastic analysis, von Mises.

1. INTRODUCTION

Wheel and rail in service undergo continual wear, so that in time all wheel profiles will be different. It is generally accepted that important damage phenomena as wear, rolling contact fatigue and especially head checks, result from overloading the rail material, [7, 8].

The optimization by grinding a worn rail profile requires the development of a software to reconstruct a rail profile using circular arcs but also to minimize contact stresses.

The wheel-rail geometry dramatically influences the rates of wheel-rail wear and contact fatigue. In the wheel-rail contact, the separation between the contacting surfaces depends on a lot of variables as: wheel profile, rail profiles, rail inclination, track gauge, inside gauge and lateral shift of the axle. Figure 1 points out the longitudinal and lateral creepages accompanying the main rolling loading.

The main criterion which exists to judge whether or not the stresses occurring for a wheel rolling over a rail are critical is the shakedown map, firstly introduced by Johnson K. L. [5].
If the load was higher than the elastic limit than at the first wheel passage, the plastic deformation occurs and a residual stress state results after unloading. For the next successive passages of the load, the material exhibits an elastic evolution being able to bear higher loads without additional plastic deformations. For the particular case of line contact the result is pointed out in the shakedown map, figure 2, whereby the presence of sliding has been considered. The elastic limit, the shakedown limit for elastic-perfectly plastic as well as for kinematic hardening material are given in the diagram (fig. 2).

If the shakedown limit was passed, then continuous plastic deformations are unavoidable. If the traction coefficient was sufficiently small \((T/N < 0.25)\) plastic deformations occur in the bulk, whereas for higher values of the traction coefficient, the largest plastic deformations are at the contact surface.

\[ \text{Fig. 2. Shakedown map for the line contact, [5].} \]

Continuous plastic deformation at the surface, mainly at the gauge corner of the high rail in a curve, results in ratcheting initiation of cracks and finally head-checks development. The influence of both plastic deformation (ratcheting) and wear should be managed synergistically with other system parameters. Unfortunately, the rail material has a constitutive law different from the one considered by Johnson in figure 2. Simulations able to provide shakedown maps for the rail material have not been published up to now.

A Stribeck diagram points out the dependency between the friction coefficient and the relative velocity. In this respect, figure 3 presents the Stribeck curves for 3 types of interfacial layers (water, oil and grease) [9].

\[ \text{Fig. 3. Stribeck curve for a wheel–rail contact, [9].} \]

\[ \text{Fig. 4. Traction curve for a wheel–rail contact, [9].} \]

2. ANALYTICAL FORMULATIONS

A hypothetical rectangular contact area denoted \(A_h\) is built on the common tangent plane, around the initial contact point. The hypothetical contact area is chosen large enough to overestimate the unknown real contact area, \(A_h \geq A_j\) (fig. 5).

\[ \text{Fig. 5. The real and virtual contact areas.} \]

A Cartesian system \((x, y, z)\) is introduced, its \(x-O-y\) plane being the common tangent plane and with its origin located at the left corner of the hypothetical rectangular area. The elastic deflection of each surface is measured in the direction of the corresponding outer normal and is denoted by \(w_I(x, y)\) and \(w_{II}(x, y)\), respectively. The sum of the individual deflections at any generic point \((x, y)\) is defined as a composite deflection, denoted by \(w(x, y)\).

The model of surface deformation is defined by the following three equations:

1. the geometric equation of the elastic contact:
   \[ g(x, y) = h(x, y) + n(x, y) - \sigma_0 \]
The integral equation of the normal surface
displacement, (Boussinesq formula):
\[ w(x,y) = \frac{1}{\pi} \left( \frac{1-v_x^2}{E_{II}} + \frac{1-v_y^2}{E_{II}} \right) \int_{\alpha} \frac{\rho(x,\eta)}{\sqrt{(x-\xi)^2+(y-\eta)^2}} \, d\xi \, d\eta \] (2)
where: \( E_{II}, v_x, v_y, \) are Young’s modulus and
Poisson’s ratio for each of the elastic materials, and
\( p(x,y) \) is the interfacial contact pressure:
c) the load balance equation:
\[ \int_{A} p(x,y) \, dx \, dy = Q \] (3)
where \( Q \) is the applied normal force.

The constraint equations of non-adhesion and
non-penetration must be fulfilled:
\[ g(x,y) = 0, \quad p(x,y) > 0, \quad (x,y) \in A \] (4)
\[ g(x,y) > 0, \quad p(x,y) = 0, \quad (x,y) \notin A \] (5)

Hooke’s law is used to obtain the stress tensor:
\[ \sigma_{ii} = \frac{E}{1+v} \left[ u_{i,i} + \frac{v}{1-2v} d\nabla U \right] \] (6)
\[ \tau_{ij} = \frac{E}{1+v} (u_{i,j} + u_{j,i}) \] (7)
where \( U \) is the displacement vector.

For non-Hertzian concentrated contacts there is not available an analytic solutions for the
displacement vector \( U \), so that a discrete presentation of the problem becomes compulsory.

3. NUMERICAL FORMULATION

A uniformly spaced rectangular array is built on
the hypothetical rectangular contact area with the grid
sides parallel to the \( x \) and \( y \)-axes, figure 5. The nodes
of the grid are denoted by \((i, j)\), where indices \( i \) and \( j \)
refer to the grid columns and rows, respectively. In
the considered Cartesian system, the coordinates of
the grid node \((i, j)\) are denoted by \((x_i, y_j)\) and are
given by \( x_i = i \cdot \Delta x , \quad (0 \leq i < N_x) \) and \( y_j = j \cdot \Delta y , \quad (0 \leq j < N_y) \) where \( \Delta x \) and \( \Delta y \) are the grid spaces in the \( x \) and \( y \)-directions, respectively.

The real pressure distribution is approximated
by a virtual pressure distribution, a piecewise-
constant approximation between grid nodes being
typically used. The analytic formulation, represented
by equations (1) - (5) is replaced by the discrete
equations (8) - (12):
\[ g_{ij} = h_{ij} - R_{ij} + w_{ij} - \delta_0 \] (8)
\[ w_{ij} = \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} K_{i-k, j-l} p_{kl} \] (9)
\[ \Delta x \Delta y \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} p_{ij} = F \] (10)
\[ g_{ij} = 0, \quad p_{ij} > 0, \quad (i, j) \in A \] (11)
\[ g_{ij} > 0, \quad p_{ij} = 0, \quad (i, j) \notin A \] (12)

The influence function \( K_{ij} \) describes the deformation
of the meshed surface due to a unit pressure
acting in element \((k, l)\). The coefficients \( K_{ij} \) are found
by integrating the Boussinesq equation for a patch
load represented by a unit pressure acting normally
over the element of area \( \Delta x \Delta y \).
\[ K_{ij} = \frac{1}{\pi} \left( \frac{1-v_x^2}{E_{II}} + \frac{1-v_y^2}{E_{II}} \right) \int_{\eta_{ij}} \frac{1}{\sqrt{(x-\xi)^2+(y-\eta)^2}} \, d\xi \, d\eta \] (13)
The integral in Eq. (13) was evaluated by Love,
Johnson K. [6] to give,
\[ K_{ij} = \frac{1}{\pi} \left( \frac{1-v_x^2}{E_{II}} + \frac{1-v_y^2}{E_{II}} \right) f(x,y) \]
where the function \( f(x,y) \) has the form:
\[ f(x,y) = x \ln \left( y + \sqrt{x^2+y^2} \right) + y \ln \left( x + \sqrt{x^2+y^2} \right) \] (14)
and \( x1 = x_i - \Delta x / 2, \quad x2 = x_i + \Delta x / 2, \quad y1 = y_j - \Delta y / 2, \quad y2 = y_j + \Delta y / 2. \)

The components of the stress tensor induced in the
point \( M(x,y,z) \) are obtained by superposition:
\[ \sigma_{ij}(x,y,z) = \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} C_{ijkl} p_{kl} \] (16)
where the influence function \( C_{ijkl}(x,y,z) \) describes
the stress component \( \sigma_{ij}(x,y,z) \) due to a unit
pressure acting in patch \((k, l)\). That is a Neumann type
problem of the elastic half-space theory. Closed form
expressions can be found in [1, 4].

4. SOLVING ALGORITHM

The Conjugate Gradient Method, (CGM), with
the iterative scheme proposed by Polonsky and Keer,
[10, 11], has been chosen to solve the mentioned
algorithmic system of equations. In order to increase
the efficiency of the numerical algorithm, a dedicated real
discrete fast Fourier transform routine for 3D contact
problems has been developed and incorporated into
the code, Cretu [1, 2].

The material was considered as having an
elastic-perfect plastic behaviour that requires to each component of the pressure vector to fulfill the
supplementary constraint:
\[ p_{ij} \geq p_I \Rightarrow p_{ij} = p_I \] (17)
where \( p_I \) is the value of the pressure able to initiate
the plastic yield.

5. CONTACT GEOMETRY

For the case of wheel-rail contact, the separation
\( h(x,y) \) between the contacting surfaces depends
on a lot of variables as: wheel profiles, rail profiles,
rail inclination, track gauge, inside gauge and lateral
shift of the axle, figure 6.
The Brent’s method has been incorporated into the computing scheme to find, for the unloaded conditions, the first contact point of the two surfaces. The Brent’s method combines root bracketing, bisection, and inverse quadratic interpolation to converge from the neighborhood of a zero crossing, [12]. The final form for the separation \( h(x, y) \) was found as follows:

\[
h(x, y) = zw(y) + rw(y) - \sqrt{rw(y)^2 - zw(x)^2 - zr(y)}
\]

(18)

where: \( zw(y) \) is the wheel profile at the coordinate \( y \); \( rw(y) \) is the wheel radius at coordinate \( y \); \( zw(x) \) is the wheel profile at coordinate \( x \); \( zr(y) \) is the rail profile at coordinate \( y \).

a. Material properties and load:
- Elasticity (Young) modulus: \( 2.1 \times 10^5 \) [MPa];
- Poisson ratio: 0.28;
- external normal load \( 10^5 \) [N];

b. Wheel and rail (track) related:
- wheel profiles: S1002 (fig. 7);
- rail profiles: UIC60, (fig. 8).
- rail inclination: 1/40;
- track gauge: 1435 [mm];
- inside gauge: 1360 [mm];
- wheel radius: 460 [mm];
- lateral shift of the axle: 0 [mm];
- yaw angle: 0°;
- roughness amplitude: 0.0 [µm].

6. VON MISES EQUIVALENT STRESS

The yield limit considered in the condition (17) was \( \sigma_y = 580 \) MPa, that corresponding to a wheel manufactured from R7T steel grade, as it is prescribed by UIC812-3 regulation. The pressure distribution is presented in figure 11 for the case of a pure normal load condition.
Von Mises equivalent stress has been chosen to evaluate the intensity of the loading regime, figure 12 to figure 21.

a) Pure normal load,
- figure 12 in the plane xOz;
- figure 13 in the plane yOz.

b) Combined loads:
- figure 14, in the plane xOz, with \( f_x = -0.250 \);
- figure 15, in the plane xOz, with \( f_x = -0.125 \);
- figure 16, in the plane xOz, with \( f_x = 0.125 \);
- figure 17, in the plane xOz, with \( f_x = 0.250 \);
- figure 18, in the plane yOz, with \( f_y = -0.250 \);
- figure 19, in the plane yOz, with \( f_y = -0.125 \);
- figure 20, in the plane yOz, with \( f_y = 0.125 \);
- figure 21, in the plane yOz, with \( f_y = 0.250 \).
7. CONCLUSIONS

1. Wheel and rail in service undergo continual wear, so that in time all wheel profiles will be different. The optimization by grinding a worn rail profile requires the development of a software to reconstruct a rail profile using circular arcs but also to minimize contact stresses.

2. A fast solver has been developed to obtain the 3D pressure distribution in non-Hertzian wheel-rail contacts subjected to both normal loads and tangential frictional forces. The pressure distributions have been obtained considering both the elastic and the elastic-perfect plastic material.

3. The main change induced by the existence of a combined load is a slight increase of the maximum value of the von Mises stresses, this one being attained at smaller depth.

4. In respect to elastic solution, the elastic-perfect plastic analysis provides smaller values for the maximum von Mises stress, but maintains the mentioned influence of the tangential friction force.

5. The elastic-plastic analysis model and the corresponding computer code proved to be an efficient tool for building the missing shakedown map for the common rail material, as well as for further investigations regarding the influence of the rail wear on pressure distribution in worn wheel-rail contacts.

REFERENCES