KINEMATIC AND DYNAMIC ASPECTS OF DAMPED COLLISION BETWEEN TWO METALLIC BALLS

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ABSTRACT

Recently, a new model of the behaviour of a collision has been proposed by Flores. The model describes the behaviour during the impact of a complete range of metallic materials, from the soft to the hard ones.

The present paper outlines the influence of the coefficient of restitution upon several kinematic and dynamic parameters of collision.

Keywords: Multibody impact, nonlinear dynamics, coefficient of restitution, hysteresis loop

1. INTRODUCTION

The impact or collision phenomenon can be described as the mechanical phenomenon characterised by a sudden variation of kinematical parameters of the system it takes upon. The rapid change of kinematical parameters leads to the occurrence of high intensity tensions in the system. The phenomenon is extremely complex. The achievement of a comprehensive study of impact phenomenon is practically impossible. Consequently, a series of simplifying hypothesis are made in the scientific literature.

As a first aspect, the propagation of elastic waves resulting from the impact phenomenon is considered. Therefore, a first classification of the collisions occurs, these being ordered as:

- high-speed collisions, when the velocities of the bodies are greater than the speed of sound, \( c_0 = \sqrt{E/\rho} \), in the materials of which the bodies are made (\( E \) is the Young modulus and \( \rho \) is the density of the material). For this case, the materials are subjected to large plastic deformations and the bodies penetrate one into another, like projectiles;
- low-speed collisions, when the speeds are much smaller than \( c_0/1000 \), these being characteristic for running the typical mechanical systems.

Two approaches were distinguished for the study of collisions:

a) the phenomenon is considered to take place instantaneously, the bodies are hypothetically rigid and the characteristic collision parameter is considered the coefficient of restitution, \( e \) [1]. The inconvenience of this approach resides in the fact that the impact forces arising during the contact of the bodies are impossible to estimate;

b) the impact phenomenon is considered to take place during a finite period of time, having a continuous evolution in time, and the colliding bodies are supposed deformable.

In the case of a collision, two phases are identified: an approaching phase, during which the bodies come into contact and approach till the distance between them becomes minin, and therefore, at this moment, the relative velocity is zero. Due to the elastic properties of the bodies, after the approaching phase, the separation or detaching phase arrives, during which the bodies remove until the complete separation happens.

2. THE COEFFICIENT OF RESTITUTION COR

Both methods used in the study of collision, the one considering the instantaneous collision and the method of finite impact time phenomenon, use, for global characterization of the behaviour, the so called coefficient of restitution, COR. For the case when the collision is regarded as instant, the coefficient of restitution is defined as the ratio, taken with changed
sign, between the final relative velocity \( v_f \) of the contact points and the initial relative velocity, \( v_i \).

\[
e = - \frac{v_f}{v_i} \quad (1)
\]

In the case when a continuous variation of the kinematical and dynamical parameters is considered, by denoting the normal approach between the two bodies with \( \delta \), one can notice that the approach starts at the moment \( t_i \) and increases up to a maximum value \( \delta_{\text{max}} \) reached at the moment \( t_{\text{max}} \) when the relative velocity \( \dot{\delta} \) of the contact points is zero. Then the detaching phase follows, during which the bodies separate, phase that goes on until the moment \( t_f \) when the complete separation takes place. In this case the restitution coefficient can be defined in two ways. Thus, according to Newton [2, 3], the coefficient of restitution is defined as the ratio between the projections on the common normal in the contact point of the final and the initial relative velocity, respectively:

\[
e = - \frac{v_f \cdot \mathbf{n}}{v_i \cdot \mathbf{n}} \quad (2)
\]

A different manner in defining the coefficient of restitution belongs to Poisson, [4], [5], as the ratio between the projections on the normal direction of the percussion from detaching phase and the percussion from the approaching phase.

\[
e = - \frac{\int_{t_i}^{t_f} F dt}{\int_{t_i}^{t_{\text{max}}} F dt} \quad (3)
\]

where \( F \) is the interaction force during collision.

Unlike the last definition modality for the coefficient of restitution, the first way of definition is more convenient to apply as it does not require writing dynamic equations for each of the two collision phases. The difficulty of using the coefficient of restitution according to Newton’s hypothesis was observed by Kane [6], who showed that in certain situations, Newton’s definition leads to violation of the law of conservation of energy. It is required to identify the situations when the two definitions are equivalent.

3. THEORETICAL MODELS OF ELASTIC COLLISIONS

In the case of disregarding the internal damping, Timoshenko [7] gives the model for centric collision of two spheres. Applying this model, the variation in time of the force and normal approach and implicitly the maximum force and the contact duration can be estimated. Both the force and the normal approach present similar variations during the two collision phases.

In the case of considering the internal friction, the most convenient model is the Kelvin-Voigt model [8]. The model consists of a mass \( m \) concurrently attached to an elastic element, of \( k \) elastic coefficient, and a linear damper having the damping factor \( \mu \). The model presents the advantage that its behaviour is described by a linear ordinary differential equation, described by:

\[
m\ddot{x} + \mu \dot{x} + k x = 0 \quad (4)
\]

for which the plot presents a hysteresis loop (Fig.1).

The inconvenience of the model consists of the open loop, this indicating, for the last part of the collision, the presence of the attraction forces.

This aspect was noticed by Dubowsky and Freudenstein [9, 10]. Hunt and Crossley [11], showed that, for the Kelvin–Voigt model, if the damping and elastic coefficients are chosen as variable parameters depending on the normal approach, a model presenting closed hysteresis loop is obtained. Based on this remark, Lankarani and Nikravesh [12], obtained the following nonlinear differential equation:

\[
x \left( t \right) = \sqrt{1 - \frac{3(1 - e^2)}{4} \frac{\dot{\delta}}{\dot{\delta}_{\text{max}}}}. \quad (5)
\]

where \( K \) is a coefficient considering the elastic characteristics of the bodies and their geometry in the vicinity of contact regions and \( \dot{\delta}_{\text{max}} \) is the initial impact speed. The equation (5) is a nonlinear ordinary nonhomogeneous equation. The boundary conditions are:
\[ \dot{\delta} = \delta^{(-)}, \text{ for } t = 0, \]
\[ \dot{\delta} = 0, \text{ for } t = t_{\text{max}}. \]  

(6)

The numerical integration of the equation is difficult due to the form of the equation and to the unknown value of \( t_{\text{max}} \). The authors found their own integration method. The validation of integration accuracy is proved by the Lankarani’s observation, \[12\], that the value of coefficient of restitution, \( e_{\text{out}} \), resulting from solving the equation (5) (Fig. 2), differs from the initial coefficient of restitution considered. In Figure 2, the dependence \( e_{\text{out}} = e_{\text{out}}(e_{\text{in}}) \) obtained by the authors, is represented, where \( e_{\text{in}} = e \) is the initial coefficient of restitution. The plot is identical to the graph presented by Lankarani [12]. The model described by equation (5) can be applied only for the collisions of hard bodies, for which \( e > 0.85 \).

Recently, Flores [13], using the energetic hypothesis, reached the conclusion that in the displacement-velocity plane, the motion of the characteristic point for a current system position should be on an ellipse. Applying this result, Flores obtains the equation describing the dynamic behaviour of the system in the form:

\[ F = K \delta^n \left[ 1 + \frac{8(1-e) \delta}{5e} \right]. \]  

(7)

The equation (7) is of the same type as equation (5) and has the same initial conditions.

4. RESULTS FOR IMPACT OF TWO SPHERES MODELED BY FLORES EQUATION

The model described by equation (7) is considered, applied for the centric collision of two spheres, with initial velocity \( \delta^{(-)} = 1 \text{ m/s} \) and having the masses \( m_1 = m_2 = 1 \text{ kg} \). The influence of the coefficient of restitution upon different collision aspects is subsequently presented.

The integration of the equation (7) is based on an original method proposed by the authors and it is attained in two steps:

- first, an analytical one, for obtaining the dependence between velocity and displacement;
- secondly, a numerical one, when the dependence between normal approach and time is found.

Using the results of integration in the equation (7), there were represented the most significant plots, which describe the behaviour of the system during the impact. There are plotted the hysteresis loop, Fig. 3, and the variation with time for contact force (Fig. 4), normal approach (Fig. 5) and the approaching velocity (Fig. 6).

Fig. 2. The variation of the coefficient of restitution

\[ e_{\text{out}} = e_{\text{out}}(e_{\text{in}}) \]

Fig. 3. The hysteresis loops
The dependence with COR is represented for:
- the maximum impact force (Fig. 7);
- the approaching time and the separation time (Fig. 8);
- the damping work (Fig. 9);
- the coefficient of restitution from Newton hypothesis (Fig. 10);
- the coefficient of restitution from Poisson hypothesis (Fig. 11).

Fig. 4. Plots of the contact force variation versus time

Fig. 5. The normal approach variation versus time

Fig. 6. Plots of the velocity variation in time

Fig. 7. Maximum contact force variation versus COR
A number of kinematic and dynamic aspects concerning the centric impact of two identical metallic balls were obtained using a recent model developed by Flores [13].

The hysteresis loops, closed in the origin, were obtained for different values of coefficient of restitution and the damping work increases while decreasing COR.

The damping work (the area of hysteresis loop) decreases while increasing COR.

The maximum contact force is practically the same as the Hertzian contact force for COR greater than 0.5 and increases rapidly for COR less than 0.5.
The approaching time increases slightly with COR and the detaching time drastically decreases with COR.

For COR having values in the vicinity of zero, the separation time tends to infinity and the separation velocity is zero as it models the plastic collision.

For the studied model, the values of COR obtained using the Newton hypothesis (kinematic method) or the Poisson hypothesis (dynamic method) are the same.

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